

## FREE CONVECTIVE HEAT TRANSFER TO SUPERCRITICAL CARBON DIOXIDE\*

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(Received 7 June 1980)

**Abstract** — Experiments on free convective heat transfer from electrically heated platinum wires and a platinum strip to supercritical carbon dioxide were performed for a wide range of bulk conditions. It is shown that heat transfer can be predicted by a conventional Nusselt-type correlation if the dimensionless numbers are based on integrated thermophysical properties in order to account for large changes in these properties. The anomaly of thermal conductivity has to be considered. Agreement between the experimental results and the correlation is within 10% even for very thin wires when for those a correction factor is introduced.

### NOMENCLATURE

$c$ ,	specific heat capacity;
$d$ ,	wire diameter;
$g$ ,	gravitational constant;
$h$ ,	heat transfer coefficient;
$h^*$ ,	specific enthalpy;
$l$ ,	wire (strip) length;
$p$ ,	pressure;
$r$ ,	co-ordinate normal to surface;
$t$ ,	temperature [ $^{\circ}\text{C}$ ];
$T$ ,	thermodynamic temperature [K];
$\beta$ ,	coefficient of thermal expansion;
$\lambda$ ,	thermal conductivity;
$\eta$ ,	dynamic viscosity;
$\nu$ ,	kinematic viscosity;
$\rho$ ,	density;
$Gr$ ,	Grashof number;
$Gr_i$ ,	$\frac{gd^3}{\nu_i^2} \frac{\rho_b - \rho_i}{\rho_i}$ ;
$Nu$ ,	Nusselt number;
$Nu_i$ ,	$= \frac{hd}{i}$ ;
$Nu^*$ ,	modified Nusselt number with $\Delta\lambda$ [equation (3)] omitted;
$Pr$ ,	Prandtl number;
$Pr_i$ ,	$2 \frac{\eta_i h_i^* - h_b^*}{\lambda_i t_w - t_b}$ ;
$Ra$ ,	Rayleigh number;
$Ra_i$ ,	$Gr_i Pr_i$ ;
$Ra^*$ ,	modified Rayleigh number with $\Delta\lambda$ [equation (3)] omitted.

### Subscripts

$b$ ,	bulk condition;
$c$ ,	at critical state;
$i$ ,	integrated mean value, equation (4);
$p$ ,	at constant pressure;
$pc$ ,	pseudocritical (temperature);
$w$ ,	heater wall.

### 1. INTRODUCTION

THE INCREASING number of applications of heat transfer to supercritical fluids near their thermodynamical critical point has revived the interest in investigating this particular heat transfer problem in the last two decades. In modern power plants heat is transferred to supercritical water. Rockets are cooled with supercritical fuel and superconductivity effects are achieved by cooling the conductor with fluids which are close to their critical points. Also highly charged machine elements like gas turbine blades and computer elements are cooled with supercritical fluids.

One of the earliest experimental investigations done on heat transfer to a critical fluid is that of Schmidt *et al.* [2]. They found that the free convective heat transfer coefficient increases considerably as the liquid approaches its critical state in a closed loop. This was attributed to the large changes of the thermophysical properties with temperature in the critical region: the coefficient of thermal expansion and the specific heat capacity strongly increase, the dynamic viscosity decreases. Meanwhile it is known that also the thermal conductivity increases in the vicinity of the critical point. All these changes improve the mechanism of heat transfer and particularly that of free convection. A single closed pipe filled with carbon dioxide so that a near critical state is reached within this 'heat pipe' exhibits increases in efficient thermal conductivity up to 12000 times the conductivity of copper [3].

It is difficult to solve free convection problems analytically because the temperature and the velocity fields depend upon each other. In the critical region the integration of the governing differential equations is aggravated by the fact that the thermophysical proper-

\*The material presented here is taken from a Dr.-Ing. thesis submitted to the Universität Stuttgart by R. J. Neumann — bibliography reference [1].

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ties are extremely temperature dependent. Many authors used a modified substitution method developed by Ostrach [4] and Sparrow and Gregg [5] to integrate the differential equations.

In most cases the evaluation was for heat transfer from a flat plate. Reimann and Grigull [6] solved the problem for the free convective heat transfer from thin wires. The agreement of the numerical solutions with the experimental results obtained by all these authors is fairly good only when the fluid is not in a near critical state. Deviations in the near critical state may be due to the neglect of the anomaly in thermal conductivity near the critical point, in the older works.

In calculating heat transfer coefficients it is customary to use Nusselt correlations. For free convection flow with dynamic forces being negligible compared to viscous forces

$$Nu = f(Ra) \quad (1)$$

where

$$Ra = Gr \cdot Pr; \quad Gr = \frac{gd^3 \beta \Delta t}{\nu^2}; \quad Pr = \frac{\eta c_p}{\lambda} \quad (2)$$

Rigorously, a function of this kind can only be applied to solve physically similar heat transfer problems. This means that the velocity distribution is in a fixed relation to the temperature distribution, corrected only by the Prandtl number. This holds when the physical properties in equation (2) are not temperature dependent. It has been tried to determine Nusselt correlations for free convection heat transfer near the critical point by evaluating the thermophysical properties at an arithmetic mean temperature and introducing a suitable correction term. This method works when the critical or pseudocritical state in the fluid is not approached too closely ( $\pm 2$  K, e.g. chapter 5).

In the early fifties Goldmann [7] found that heat transfer to supercritical water by forced convection is sometimes accompanied by a noise like a 'whistle' and by an increase in the heat transfer coefficient. He postulated a special heat transfer mechanism which was called by him 'pseudoboiling'. Since then similar phenomena have been observed at free convection from electrically heated wires to supercritical carbon dioxide by Knapp and Sabersky [8], Hahne *et al.* [9] and Nishikawa *et al.* [10]. The increase in heat transfer started with oscillations of the wire at temperature differences not less than 70 K. No satisfactory explanation for this unusual phenomenon has been published yet.

The experiments described in this paper were designed to find a correlation for heat transfer by free convection from thin wires and a flat strip to supercritical carbon dioxide near the critical point. For this purpose, the extreme temperature dependence of the thermophysical properties is taken into account. Experiments on pseudoboiling phenomena were performed and are reported in [1]. They exhibited effects

of the thermal diffusivity and the geometry of the heating surface.

## 2. THERMOPHYSICAL PROPERTIES

For carbon dioxide the following critical properties were taken:  $t_c = 31.04^\circ\text{C}$ ,  $p_c = 71.81$  bar and  $\rho_c = 468$  kg/m<sup>3</sup>. Figure 1 shows that density  $\rho$  and enthalpy  $h^*$  are extremely temperature dependent in the critical region, particularly in the vicinity of the critical point. As a consequence, the coefficient of thermal expansion  $\beta$  and the specific heat capacity  $c_p$  respectively, show a sharp peak at certain temperatures called therefore pseudocritical temperatures,  $t_{pc}$ . The pseudocritical temperature  $t_{pc}$  increases with pressure (Table 1). The peaks in  $c_p$  and  $\beta$  tend to infinity at the critical point and become smaller as the pressure increases. All thermodynamic properties used here were calculated with the canonical equation of state for carbon dioxide [11].

The dynamic viscosity  $\eta$  [12] is also shown in Fig. 1. It decreases with temperature, proportionally to density. The thermal conductivity  $\lambda$  shows a sharp peak at the pseudocritical temperature.

Although no explanation has been found for this strange behaviour its existence is commonly accepted. The thermal conductivity can be expressed as follows

$$\lambda = \lambda(T, \rho = 0) + \bar{\lambda}(\rho) + \Delta\lambda. \quad (3)$$

The first two terms represent the well-known dependence of a transport property on temperature and density. The third term  $\Delta\lambda$  accounts for the abnormal peaks in the vicinity of the pseudocritical tempera-

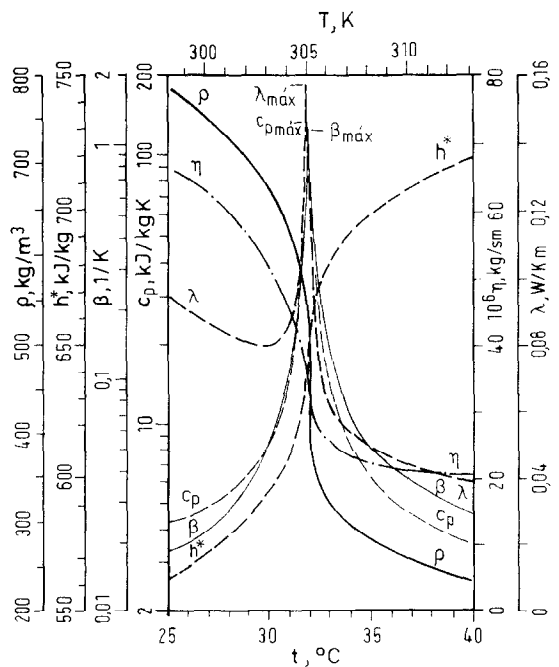


FIG. 1. Thermophysical properties of carbon dioxide at 75 bar.

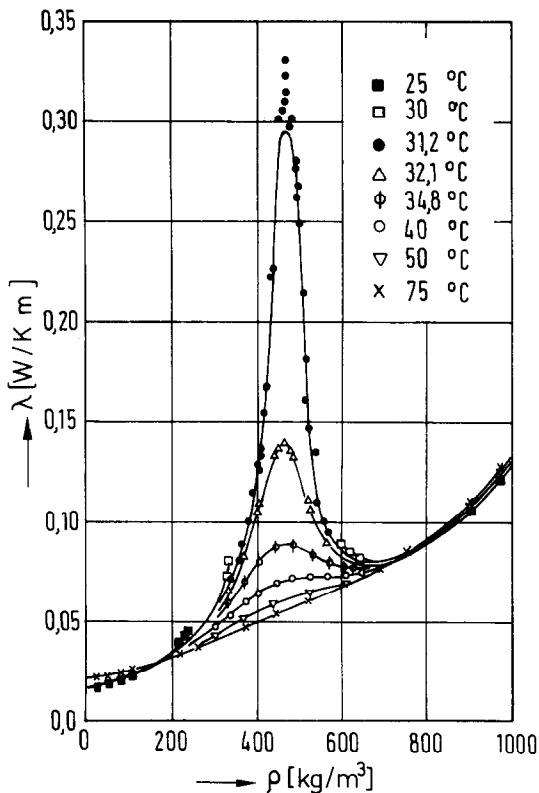


FIG. 2. Thermal conductivity of carbon dioxide at various temperatures, measured and calculated according to [13].

tures. Le Neindre *et al.* [13] proposed that this term be evaluated by means of the 'scaling laws' or as a function of the compressibility. Both methods were applied and showed that for densities greater than approximately  $260 \text{ kg/m}^3$  the solution obtained with the scaling law method agreed better with the measurements published by the same authors, while for smaller densities the function of the compressibility was more appropriate. Figure 2 shows the theoretical solution according to equation (3) and experimental data.

### 3. EXPERIMENTAL PROCEDURE

A schematic sketch of the apparatus is shown in Fig. 3. Essentially, it consists of an electrically heated wire (strip) (c) mounted horizontally in a high pressure vessel (a). Experiments with three platinum wires of 0.05, 0.1 and 0.3 mm dia and a platinum strip of 0.0125 mm thickness and 5 mm height were performed. All these elements were 67 mm long. The pressure vessel\* has an inner diameter of 114.3 mm and 76.2 mm inner length. A glass window (f) in each end plate allows for Schlieren projections. Temperature control was achieved by immersing the entire test chamber

\* The authors gratefully acknowledge Professor Sabersky's providing this vessel. A detailed description of it is given in [8].

into a thermostatic water bath, the primary control coming from a thermostat (h). The pressure in the vessel was controlled by either changing the temperature of the fluid in the auxiliary vessel (j) or with the piston pump (k). Pressure measurements were made with two precision bourdon pressure gauges (m). The temperature of the bulk fluid was measured by three thermocouples located at various levels below the heating element; the mean temperature of the heating element was determined by measuring its electrical resistance. Platinum is appropriate for these experiences due to its high and well-known temperature coefficient of electrical resistance.

With corrections to this mean temperature obtained from the Fourier equation with inner sources the wall temperature of the heating element was calculated.

The wires and the strip were heated by applying a constant voltage source. The heat flux was calculated from the measured voltage drop in a precision resistance (o) and the heating element (c). Heat losses caused by radiation and axial conduction were negligible.

Prior to the experiments the whole system was evacuated and filled alternately several times with highly pure 99.99% carbon dioxide from the bottle (x). The final filling of the system was assisted by cooling both vessels (a and j) approximately 15 K below room temperature and opening the supply line for a few minutes, thus undercooling the entering fluid below the temperature of the saturated carbon dioxide in the bottle.

The test conditions are compiled in Table 1. For each pressure the bulk temperatures were primarily chosen as to be in the vicinity of the corresponding pseudocritical temperature. The 0.3 mm dia wire and especially the strip could not be heated over the limit listed in Table 1 because the dissipated heat had to be removed through the walls of the test vessel without altering significantly the bulk condition.

### 4. EXPERIMENTAL RESULTS

Typical results for heat transfer coefficients as a function of wall temperatures  $t_w$  are presented in Fig. 4 for a 0.1 mm dia platinum wire at 74 bar with bulk temperature  $t_b$  as parameter. The lines shown in this figure are calculated from equation (11), chapter 5 [1].

When  $t_b$  is much lower than the pseudocritical temperature  $t_{pc}$  (e.g.  $t_b = 10^\circ\text{C}; 20^\circ\text{C}; 25^\circ\text{C}$ ) the heat transfer coefficient increases gradually with the increase of  $t_w$  until  $t_{pc}$  is slightly exceeded.

With a further increase of  $t_w$ , the heat transfer coefficient decreases to a nearly constant and uniform value. When  $t_b$  draws near  $t_{pc}$  ( $t_b = 30; 31^\circ\text{C}$ ), the heat transfer coefficient and its rate of change becomes large and reaches a maximum when  $t_w \approx t_{pc}$ . Here, free convection heat transfer coefficients reach the order of magnitude of those of nucleate boiling.

For  $t_b > t_{pc}$  ( $t_b = 31.5; 35; 50^\circ\text{C}$ ) the heat transfer coefficient remains at comparatively small values

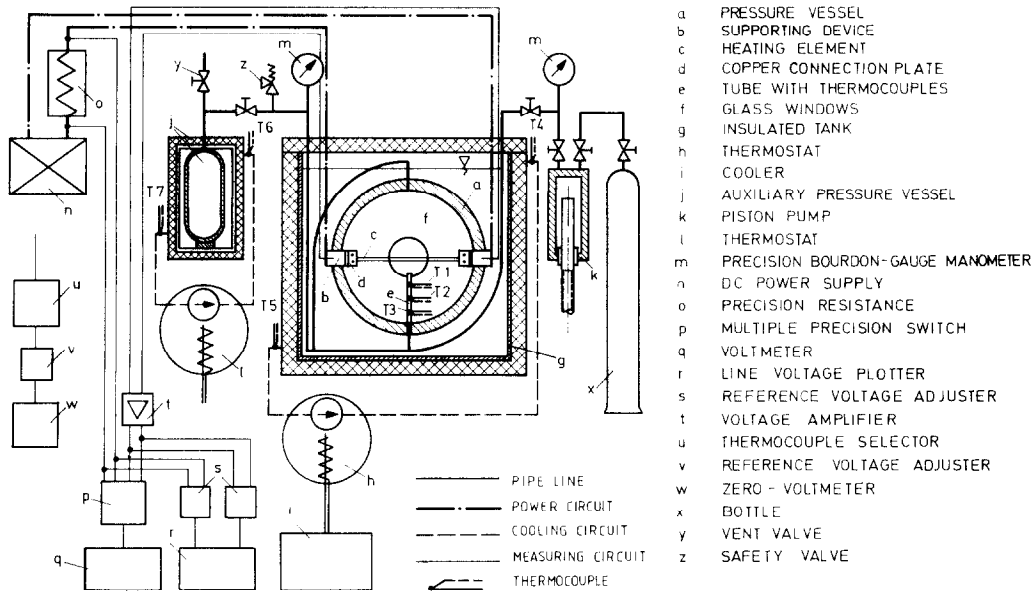


FIG. 3. Sketch of the experimental apparatus.

either slightly decreasing or being approximately constant with wall temperature. It is thus obvious that for  $t_b > t_{pc}$  the heat transfer coefficient is primarily a function of  $t_b$  while for  $t_b < t_{pc}$  it is a function of both  $t_w$  and  $t_b$ .

Figure 5 gives the heat transfer coefficient for the same wire at a bulk pressure of 80 bar. The same qualitative observations as described above can be made under these test conditions. Quantitatively the peaks of the heat transfer coefficient at  $t_w$  close to  $t_{pc}$  are smaller than before. At other supercritical pressures as listed in Table 1 similar results were obtained

Generally, the heat transfer coefficients decrease as the wire diameter increases. The lowest values were measured for the 5 mm high strip. Compared to a 0.05 mm dia wire the heat transfer coefficients for a 0.1 mm wire were about 25% lower, for the 0.3 mm and for the strip they were roughly about 30% and 60% lower respectively. Nevertheless, for all these heating elements the dependence of the heat transfer coefficient on the temperatures  $t_b$ ,  $t_w$  and  $t_{pc}$  and on pressure was confirmed.

Boiling-like phenomena were not observed with any of the platinum heating elements.

Table 1. Experimental conditions

Heating element Pt, l = 67 mm	Bulk pressures, bar (pseudocritical temp, °C)	Bulk temperatures (°C)	Wall temperatures approx (°C)
0.05 mm wire	74 ( $t_{pc} = 31.15^\circ\text{C}$ )		stepwise
	75 (31.75)	10, 25, 31,	
	80 (34.86)	31.5, 32, 35	
	90 (40.8)	50	up to 500°C
0.1 mm wire	74, 75, 80	10, 20, 25	
	85 (37.86)	30, 31, 31.5	up to 600°C
	90, 95 (44.0)	32, 35, 50	
0.3 mm wire	74, 75, 80, 90	10, 25, 31	up to 200°C
		31.5, 32, 35,	
		50	
5 × 0.0125 mm strip	74, 90	10, 25, 31, 35, 50	up to 100°C

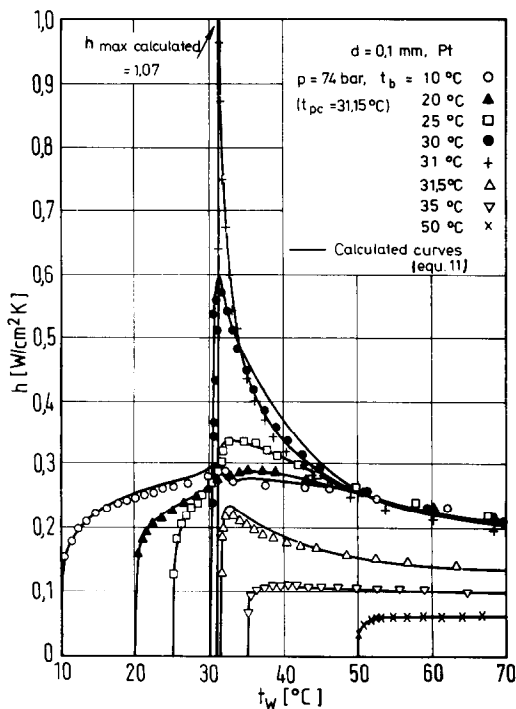


FIG. 4. Heat transfer coefficient from a 0.1 mm Pt wire at 74 bar vs wall temperature.

### 5. CORRELATIONS

Although the thermophysical properties in the critical region are extremely temperature dependent and heat transfer processes are not physically similar, it is commonly accepted to use equation (1) for the correlation of data.

A simple correlation plot,  $Nu$  vs  $Ra$ , with thermal properties taken at film temperature  $(t_b + t_w)/2$  results in anomalies for data obtained for wall temperatures near the pseudocritical state [9]: Rayleigh numbers in

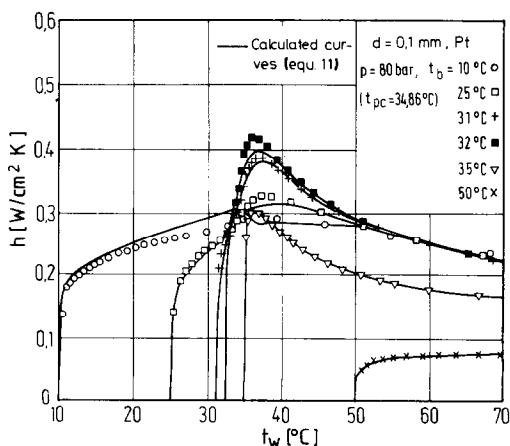


FIG. 5. Heat transfer coefficient from a 0.1 mm Pt wire at 80 bar vs wall temperature.

this state increase much more than Nusselt numbers, even for peak heat transfer coefficients, so that in the plot  $Nu$  vs  $Ra$  within a wide range of scatter a dependence  $Nu \approx \text{constant}$  is falsely pretended. This is also observed in a correlation for horizontal slots filled with  $\text{CO}_2$  [14].

In order to take account of variable properties, Nusselt [15] suggested to use an integrated mean value for any property  $P$ , as calculated from

$$P = \frac{1}{t_w - t_b} \int_{t_b}^{t_w} P(t) dt. \quad (4)$$

The application of such an integration is appropriate for any property which does not assume an infinite or an extremely large value as in the critical or pseudo-critical state. Formally, an infinite value for a property, yielding an infinite Rayleigh number would result, equation (1), in an infinite Nusselt number. By definition, however, any heat transfer requires a temperature difference. Therefore, as long as there are different temperatures (at constant pressure) there are different properties in a thermal layer adjacent to the heating surface, maintaining some resistance to the heat transport. Consequently, the heat transfer coefficient remains finite. (In our experiments, at the best, heat transfer coefficients of  $h = 9700 \text{ W/m}^2 \text{ K}$  were measured and  $h = 10700 \text{ W/m}^2 \text{ K}$  was calculated.)

The coefficient of thermal expansion and the specific heat capacity tending towards infinity in the critical state have to be especially considered in the aforementioned respect. They can be approximated as:

$$\beta = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial t} \right)_p = -\frac{1}{\rho} \left( \frac{\Delta \rho}{\Delta t} \right)_p = \frac{1}{\rho} \left( \frac{\rho_b - \rho_w}{t_w - t_b} \right)_p \quad (5a)$$

$$c_p = \left( \frac{\partial h^*}{\partial t} \right)_p = \left( \frac{\Delta h^*}{\Delta t} \right)_p = \left( \frac{h_w^* - h_b^*}{t_w - t_p} \right)_p \quad (6a)$$

These approximations have the disadvantage that the mean values only depend on the two boundary temperatures and do not take into account an extreme change of properties within the thermal layer. In order to introduce an integrated mean property into equation (5a), Kato *et al.* [16] proposed to approximate the product  $\beta \cdot \Delta t$  as

$$\beta \cdot \Delta t = \left| -\frac{1}{\rho} \frac{\Delta \rho}{\Delta t} \right|_p \cdot \Delta t = \frac{\Delta \rho}{\rho} = 2 \left( \frac{\rho_b - \rho_i}{\rho_i} \right)_p \quad (5b)$$

Our experiments showed that it is of advantage to also approximate the specific heat capacity as

$$c_p = \left( \frac{\Delta h^*}{\Delta t} \right)_p = 2 \left( \frac{h_i^* - h_b^*}{t_w - t_b} \right)_p \quad (6b)$$

with  $h_i^*$  being the integrated, equation (4), mean specific enthalpy. Neither density  $\rho$  nor enthalpy  $h^*$  assume infinite values, thus avoiding the aforementioned problem.

The factor 2 in equation (6b) stems from the assumption to attribute the integrated enthalpy to the

arithmetic mean temperature; this gives  $\Delta t = t_w - (t_b + t_w)/2$  and provides a better fit to  $c_p = dh^*/dt$ .

For the thermal conductivity the integrated mean value  $\lambda_i$  was taken as a first choice, despite the fact that it exhibits an abnormal increase close to the critical point.

With these properties, the following numbers were obtained:

$$Ra_i = Gr_i Pr_i, \quad Gr_i = 2 \frac{gd^3 \rho_b - \rho_i}{\nu_i^2 \rho_i},$$

$$Pr_i = 2 \frac{\eta_i h_i^* - h_b^*}{\lambda_i t_w - t_b}. \quad (7)$$

The Nusselt number is derived from the boundary condition

$$h(t_w - t_b) = -\lambda \left( \frac{\partial t}{\partial r} \right)_w. \quad (8)$$

This means that all the heat dissipated by the wall is transferred through a thin layer adherent to the wall, only by conduction. Thus the thermal conductivity in the Nusselt number should be evaluated at wall temperature [17]. However, due to its anomaly in the critical region, the thermal conductivity may increase considerably while — as experiments show — the heat transfer coefficient does not increase correspondingly. The experimental results proved the following Nusselt number definition to be the most suitable

$$Nu_i = \frac{hd}{\lambda_i}. \quad (9)$$

In the following analysis three different cases shall be discerned:

Case I: all temperatures are below the respective pseudocritical temperature:  $t_b < t_w < t_{pc}$

Case II: the wire temperature equals or exceeds the pseudocritical value while the bulk temperature is still

below it:  $t_b < t_{pc} \leq t_w$

Case III: all temperatures are above the pseudocritical value:  $t_{pc} < t_b < t_w$ .

Figure 6 presents  $Nu_i$  as a function of  $Ra_i$  for a 0.1 mm dia platinum wire at 75 bar and various bulk temperatures. For comparison a conventional, empirical correlation by van der Hegge Zijnen [18]

$$Nu_i = 0.35 + 0.25 Ra_i^{1/8} + 0.45 Ra_i^{1/4} \quad (10)$$

is represented as a line. Deviations of  $\pm 10\%$  to this line are also plotted in the diagram. This correlation was selected because it is based on a great number of experiments as performed by many investigators on horizontal cylinders and wires for many fluids, it covers a range of  $10^{-4} < Ra_i < 10^9$ .

The experimental data for the cases I and III fall well within the  $\pm 10\%$  scatter region of equation (10), since the physical properties do not change extremely there.

When, however,  $t_w$  approaches  $t_{pc}$ , the properties change considerably and the heat transfer coefficients reach some maximum. For this case (II) the diagram shows that  $Ra_i$  increases more rapidly than  $Nu_i$  and loops are formed outside the lower boundary of the scatter region. At high temperature differences (e.g.  $(t_w - t_b) > 80 \text{ K}$ ;  $t_b = 10^\circ\text{C}$ ) both numbers decrease again and the systematic deviations converge in the  $\pm 10\%$  region of equation (10). The largest deviations result for the lowest  $t_b$ . As  $t_b$  increases the deviations decrease; for  $t_b$  also close to  $t_{pc}$  the experimental data fall within the  $\pm 10\%$  scatter region.

At higher bulk pressures, similar but smaller systematic deviations were obtained. A comparison for three different wire diameters at the same bulk conditions is shown in Fig. 7. The deviations from equation (10) become smaller as the wire diameter increases.

Figure 8 shows the experimental data for a flat strip, simulating a vertical wall, at 74 bar and various bulk conditions. The characteristic length  $d$  in equations (7)

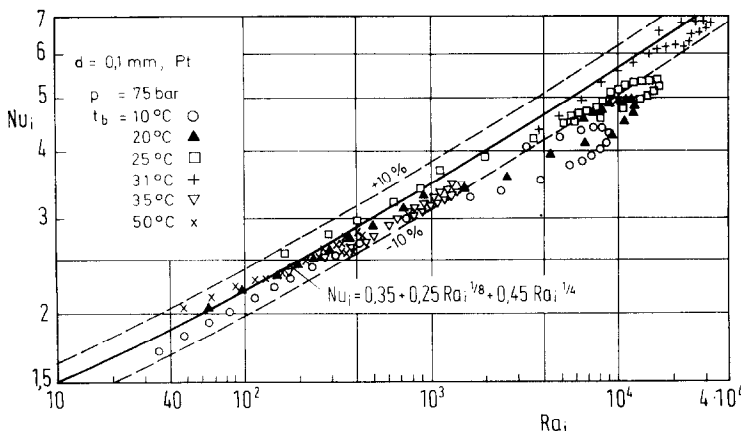


FIG. 6. Correlation of measured data from a 0.1 mm Pt wire at 75 bar.

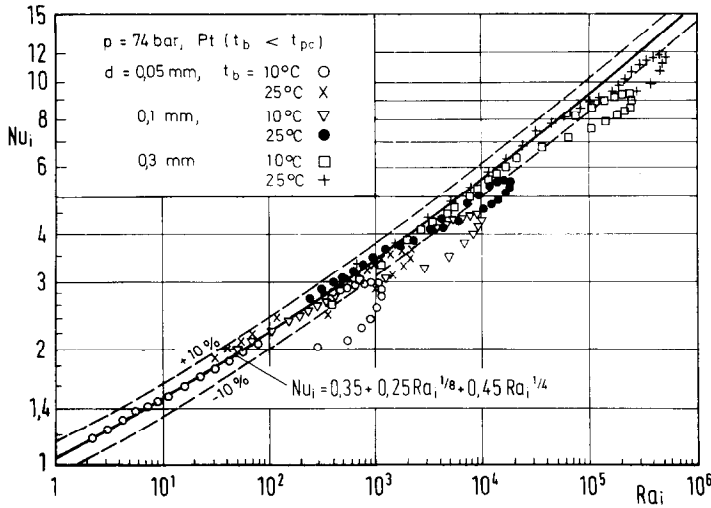


FIG. 7. Correlation of measured data from three different Pt wires at 74 bar.

and (9) was changed to the height of the strip (5 mm). Again for comparison a correlation developed by Krischer [19] for a wide range of Grashof and Prandtl numbers is plotted in the diagram. The scatter of experimental data is small confirming that a correction function has to be considered only for thin wires and case II.

In all plots the systematic deviations are largest when  $t_b$  is relatively low and  $t_w$  exceeds  $t_{pc}$  only by little (case II). Then the peak values of  $\beta$  and  $c_p$  are within the integration limits and  $Gr_t$  increases considerably. The increase of  $c_p$  is partially counterbalanced by the increase of  $\lambda_i$  within a short temperature range, causing  $Pr_t$  to increase only moderately. On the other hand, as shown in Fig. 4, the heat transfer coefficient does not

increase considerably when  $t_b$  is relatively low. Thus, as  $\lambda_i$  increases,  $Nu_i$  somewhat stagnates causing the aforementioned loops.

This suggests that the thermal conductivity may not be evaluated properly from equation (4) or the anomaly is overestimated by equation (3). Since the experimental data for small temperature differences and  $t_b$  close to  $t_{pc}$  are well represented by equation (10), however, the second possibility might be excluded.

Some authors argue that the anomaly of thermal conductivity is not significant for the heat transfer in this case. The contrary can be proven by calculating  $Nu_i$  as a function of  $Ra_i$  without considering the anomaly term  $\Delta\lambda$  in equation (3) and plotting such a correlation. This was done in Fig. 9 where now very

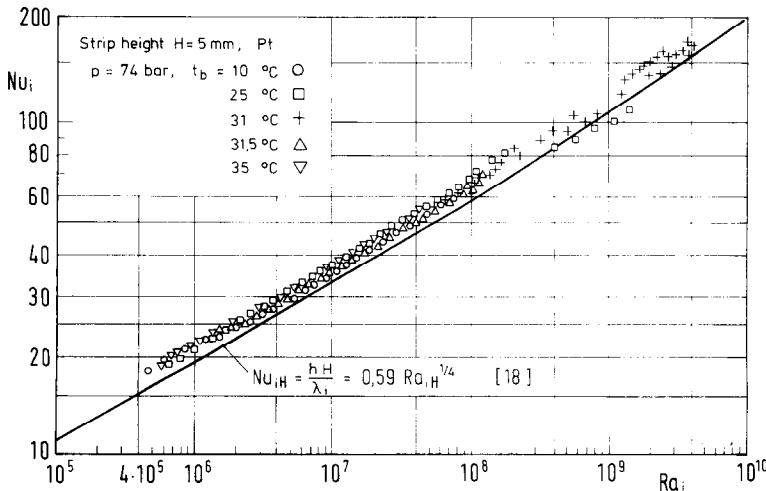


FIG. 8. Correlation of measured data from a 5 mm high platinum strip at 74 bar.

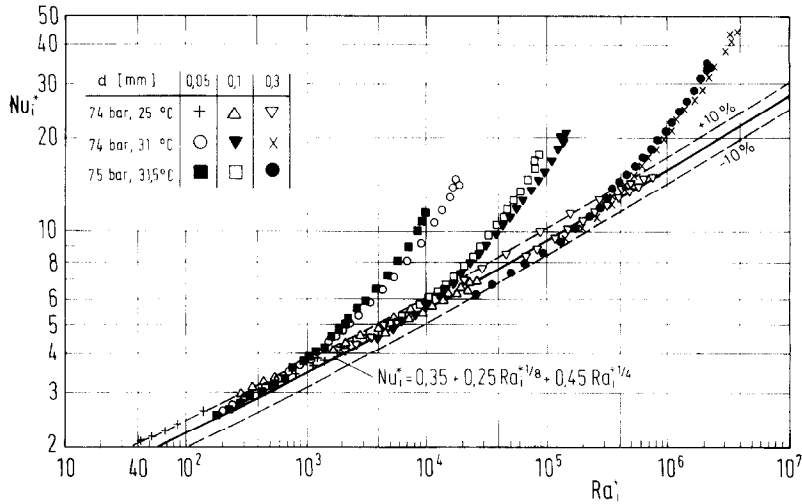


FIG. 9. Correlation of measured data for various wire diameters without considering the thermal conductivity anomaly term  $\Delta\lambda$  in  $Nu_i$  and  $Ra_i$ .

distinct deviations from equation (10) can be observed. These start when the wall temperatures approach closely the pseudocritical and they are largest when the thermal layer spans the pseudocritical state. These deviations appear for every diameter investigated here.

The finding of a decrease in deviation of data from equation (10) with increasing wire diameter (Fig. 7) suggests a special consideration for thin wires. For these, the thermal boundary layer thickness increases unproportionally compared to the diameter, and conductivity prevails in a larger region around the wire. As  $t_w$  exceeds  $t_{pc}$  at low  $t_b$  presumably only a very thin film has a high thermal conductivity, while the mean thermal resistance of the boundary layer is still high; thus the heat transfer coefficient does not increase considerably. For surfaces with a greater curvature radius, the thermal resistance due to conduction prevails only in a very thin film on the wall and convection governs the heat transfer process. The aforementioned influence of thermal conductivity is counterbalanced by the buoyancy effects.

The deviations principally depend upon  $t_w - t_{pc}$ ,  $t_{pc} - t_b$  and  $p$ . For each of these terms a separate correction function was determined from the experimental data and the following modified correlation is proposed for case II and thin wires

$$Nu_i = (0.35 + 0.25Ra_i^{1/8} + 0.45Ra_i^{1/4}(1 + f_1f_2f_3))^{-1} \quad (11)$$

where

$$f_1 = xe^{-x}, \quad x = 4.5 \sqrt{\left(\frac{T_w - T_{pc}}{T_c}\right)} \quad (12)$$

$$f_2 = \tanh\left(30 \frac{T_{pc} - T_b}{T_c}\right) \quad (13)$$

$$f_3 = 1 - 0.3 \tanh\left(15 \frac{p - p_c}{p_c}\right) \quad (14)$$

and  $T$  being the thermodynamic temperature  $T = t + 273.15$ .

This correlation (11) was used to obtain the calculated curves  $h$  vs  $t_w$  in Figs. 4 and 5.

In Fig. 10 the correlation of data according to equation (11) is shown. The fit of data as also presented in Fig. 7 is better here. All data are within a  $\pm 10\%$  scatter.

For technical use those temperature ranges were examined for which simple film temperatures  $t_f = (t_w + t_b)/2$  suffice to determine the thermophysical properties in the numbers of equation (10) and this equation still yields agreement within  $\pm 10\%$ . For the conditions listed in Table 1 these ranges are

$$t_b < t_w < t_{pc} - 2 \text{ K} \quad \text{and} \quad t_w > t_b > t_{pc} + 2 \text{ K}.$$

Thus free convection heat transfer in the critical region can be predicted from equation (10) without performing integrations as long as bulk and wall temperatures are either 2 K below or above the respective pseudocritical temperature.

## 6. CONCLUSIONS

Heat transfer coefficients for free convection heat transfer in the near critical region can be predicted by a conventional, and well-established Nusselt correlation. If a region of  $\pm 2$  K around the pseudocritical temperature is spared the thermophysical properties in the  $Nu$ - and  $Ra$  numbers may simply be based on film temperature.

Very close to the pseudocritical state where property changes are large, good agreement of experimental data and the aforementioned correlation can be obtained with integrated mean property values. The anomaly of thermal conductivity has to be taken into



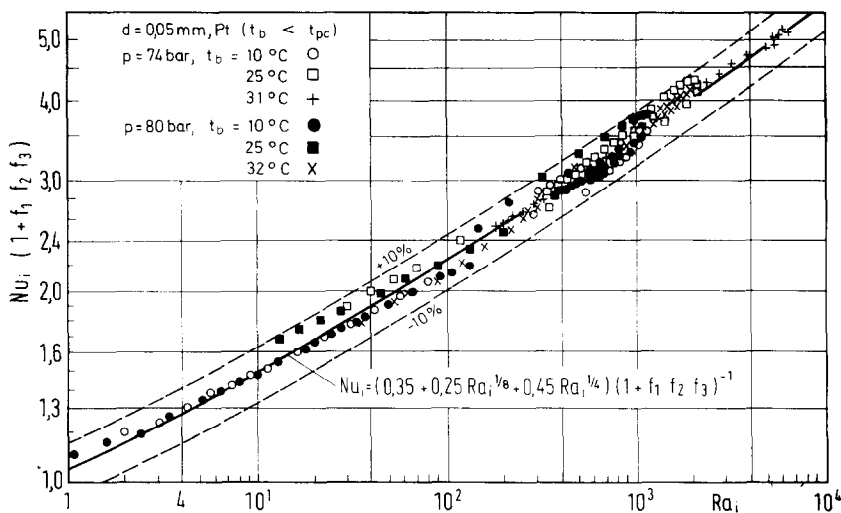


FIG. 10. Correlation of measured data according to the corrected equation (11).

account. For thin wires a correction factor introduced in the correlation yields  $\pm 10\%$  deviation. Boiling-like phenomena do not occur on platinum wires up to investigated temperatures of  $600^\circ\text{C}$ .

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#### CONVECTION THERMIQUE NATURELLE DANS LE GAZ CARBONIQUE SUPERCRITIQUE

**Résumé** — Des expériences sur la convection thermique naturelle autour de fils et d'un ruban en platine chauffés électriquement, dans du gaz carbonique supercritique, ont été conduites pour un large domaine de conditions. On montre que le transfert thermique peut être évalué par une formule du type de Nusselt si les nombres sans dimension sont basés sur des propriétés thermophysiques intégrées de façon à tenir compte des grandes variations de ces propriétés. L'anomalie de conductivité thermique est considérée. On obtient un accord meilleur que  $10\%$  entre les résultats expérimentaux et la formule, même pour les fils très fins lorsque, pour ceux-ci, un facteur correctif est introduit.

## WÄRMEÜBERGANG DURCH FREIE KONVEKTION AN ÜBERKRITISCHES KOHLENDIOXID

**Zusammenfassung** — Experimentell wurde der Wärmeübergang durch freie Konvektion von elektrisch beheizten Platindrähten und einem Platinband an überkritisches Kohlendioxid in einem weiten Bereich von thermodynamischen Zuständen untersucht. Der Wärmeübergang kann durch eine konventionelle Nusselt-Gleichung bestimmt werden, wenn die Kennzahlen mit integrierten Stoffwerten gebildet sind, um die großen Änderungen dieser Stoffwerte im kritischen Gebiet zu berücksichtigen. Die Anomalie der Wärmeleitfähigkeit muß berücksichtigt werden. Die Abweichung zwischen den experimentellen Ergebnissen und der Korrelation liegt im Bereich  $\pm 10\%$ , selbst für dünne Drähte, wenn für letztere eine Korrektur vorgenommen wird.

## СВОБОДНОКОНВЕКТИВНЫЙ ТЕПЛОПЕРЕНОС С СВЕРХКРИТИЧЕСКОЙ ОКСИ УГЛЕРОДА

**Аннотация** — Проведено экспериментальное исследование свободноконвективного переноса тепла от нагреваемых электрическим током платиновых проволочек и платиновой ленты к сверхкритической окиси углерода в широком диапазоне изменения условий в среде. Показано, что теплообмен можно рассчитать с помощью обычного уравнения подобия, содержащего число Нуссельта, если критерии подобия основаны на интегральных теплофизических характеристиках для учета их изменения в широком диапазоне. Следует также учитывать аномальное изменение теплопроводности. Экспериментальные данные совпадают с результатами расчетов с точностью до 10% даже при использовании проволочек очень малого диаметра, когда необходимо вводить поправочный коэффициент.